2.3 Cost Minimization - Practice Problems (Answers)

Ryan Safner

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Your firm can use labor l and capital k to produce output according to the production function:

$$q = 4lk$$

The marginal products are:

$$\begin{aligned} MP_l &= 4k \\ MP_k &= 4l \end{aligned}$$

Suppose you need to produce 144 units, the price of labor is \$10, and the price of capital is \$40.

1. What is the least-cost combination of labor and capital that produces 144 units of output?

Solution: Use the definition of the optimum:

$$\begin{aligned} MRTS_{l,k} &= \frac{w}{r} && \text{Definition of the optimum} \\ \frac{MP_l}{MP_k} &= \frac{w}{r} && \text{Definition of MRTS on left} \\ \frac{4k}{4l} &= \frac{(10)}{(40)} && \text{Plugging in what we know} \\ \frac{k}{l} &= \frac{1}{4} && \text{Simplifying} \\ k &= \frac{1}{4}l && \text{Multiplying both sides by} l \end{aligned}$$

So we know that we will be using 1 unit of capital for every 4 workers (this should make sense, as capital is 4 times as expensive as labor). This is the optimal ratio of inputs.

To find the exact quantities of l and k, use the production function:

$$
\begin{aligned}
q &= 4lk && \text{The production function} \\
1444l(\frac{1}{4}l) & && \text{Plugging in what we are given and what we found} \\
144 &= l^2 && \text{Multiplying} \\
12 &= l && \text{Square rooting both sides}
\end{aligned}
$$

Now that we know the quantity of labor, we can use our knowledge of the ratio of labor to capital to find the optimal quantity of capital.

$$
\begin{aligned}
k &= \frac{1}{4}l \\
k &= \frac{1}{4}(12) \\
k &= 3
\end{aligned}
$$

So using 12 workers and 3 units of capital produces 144 units of output at the lowest total cost.

2. How much does this combination cost?

Solution:

$$
\begin{aligned}
wl + rk &= C && \text{The isocost line equation} \\
10(12) + 40(3) &= C && \text{Plugging in what we know (prices) and what we found} \\
120 + 120 &= C && \text{Multiplying} \\
240 &= C && \text{Adding}
\end{aligned}
$$

The total cost of using 12 workers and 3 units of capital at current prices is \$240.

3. Does this technology experience constant returns, increasing returns, or decreasing returns to scale?

Solution:

Simply plug in combinations of labor and capital that change at the same rate, and see at what rate output changes. For example, with 1 worker, 1 unit of capital, output is:

$$q = 4lk$$
$$q = 4(1)(1)$$
$$q = 4$$

If we now *double* all inputs, so that we use 2 workers and 2 units of capital, output is

$$q = 4lk$$
$$q = 4(2)(2)$$
$$q = 16$$

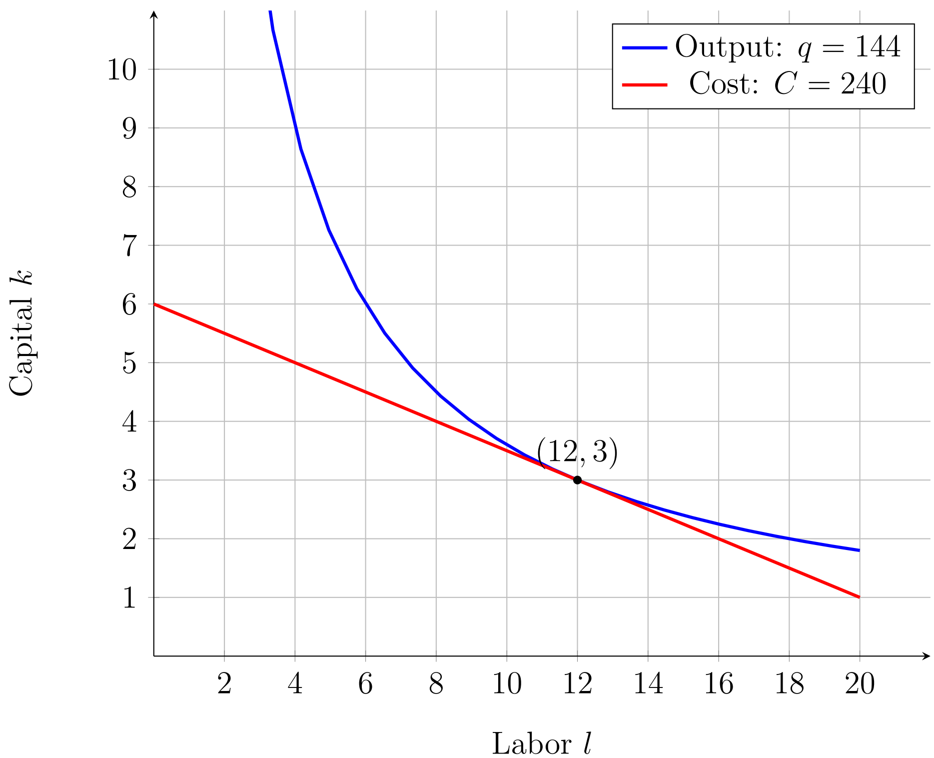
Output has quadrupled from 4 to 16, from a doubling of all inputs. Therefore, this technology exhibits increasing returns to scale.

There is a shortcut that we could use, because this function is in Cobb-Douglas format (inputs and *multiplied* by each other, and each raised to an exponent), we can simply sum the exponents:

$$q = 4l^1k^1$$

$$1 + 1 = 2$$

Because the exponents sum to a number greater than one, this technology is increasing returns. Be careful, this shortcut method *only* works for Cobb-Douglas functions!


Output: $q = 144$
Cost: $C = 240$
$(12, 3)$
Capital k
Labor l
1
2
3
4
5
6
7
8
9
10
2
4
6
8
10
12
14
16
18
20