1.6 Solving the Consumer's Problem - Practice Problems (Answers)

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ECON 306 - Fall 2019

You can get utility from consuming Soda (s) and Hot dogs (h), according to the utility function:

$$u(s,h) = \sqrt{sh}$$

The marginal utilities are:

$$MU_s = 0.5s^{-0.5}h^{0.5}$$
$$MU_h = 0.5s^{0.5}h^{-0.5}$$

You have an income of \$12, the price of Soda is \$2, and the price of a Hot dog is \$3. Put Soda on the horizontal axis and Hot dogs on the vertical axis.

1. What is your utility-maximizing bundle of Soda and Hot dogs?

Solution: Use the definition of the optimum:

$$\begin{aligned}
MRS_{s,h} &= \frac{p_s}{p_h} && \text{Definition of the optimum} \\
\frac{MU_s}{MU_h} &= \frac{p_s}{p_h} && \text{Definition of MRS on left} \\
\frac{0.5s^{-0.5}h^{0.5}}{0.5s^{0.5}h^{-0.5}} &= \frac{(2)}{(3)} && \text{Plugging in what we know} \\
\frac{0.5}{0.5}s^{(-0.5-0.5)}h^{(0.5-[-0.5])} &= \frac{2}{3} && \text{Using exponent rules for division} \\
s^{-1}h^{1} &= \frac{2}{3} && \text{Simplifying and cancelling} \\
\frac{h}{s} &= \frac{2}{3} && \text{Using exponent rules for negative exponents} \\
h &= \frac{2}{3}s && \text{Multiplying both sides by} s
\end{aligned}$$

So we know that we will be buying $\frac{2}{3}$ sodas for every 1 hot dog. This is the optimal ratio of consumption between the two goods.

To find the exact quantities of s and h, use the budget constraint:

$$\begin{aligned}
p_s s + p_h h &= m && \text{The budget constraint equation} \\
2s + 3h &= 12 && \text{Plugging in what we are given} \\
2s + 3(\frac{2}{3}s) &= 12 && \text{Plugging in what we found relating } b \text{ to } a \\
2s + 2s &= 12 && \text{Multiplying} \\
4s &= 12 && \text{Adding} \\
s &= 3 && \text{Dividing by 4}
\end{aligned}$$

Now that we know the quantity of sodas, we can use our knowledge of the ratio of sodas to hot dogs to find the quantity of hot dogs.

$$\begin{aligned}
h &= \frac{2}{3}s \\
h &= \frac{2}{3}(3) \\
h &= 2
\end{aligned}$$

2. How much utility does this provide?

Solution: How much utility do we get? Plug our optimal bundle into the utility function:

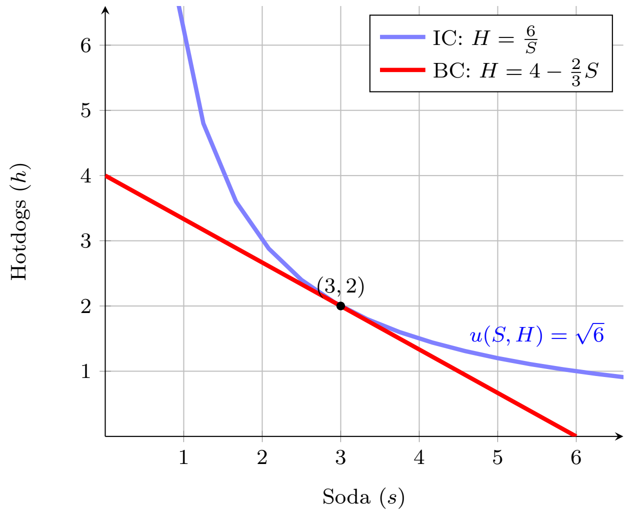
$$\begin{aligned}
u(s,h) &= \sqrt{sh} \\
u(s,h) &= \sqrt{(3)(2)} \\
u(s,h) &= \sqrt{6}
\end{aligned}$$

If we wanted to graph the indifference curve, we need to solve for h (the good on the vertical axis).

$$\begin{aligned}
u(s,h) &= \sqrt{sh} \\
\sqrt{6} &= \sqrt{sh} \\
6 &= sh \\
\frac{6}{s} &= h
\end{aligned}$$

Same with the budget constraint to graph:

$$\begin{aligned}
p_s s + p_h h &= m \\
2s + 3h &= 12 \\
3h &= 12 - 2s \\
h &= 4 - \frac{2}{3}s
\end{aligned}$$


IC: $H = \frac{6}{S}$
BC: $H = 4 - \frac{2}{3}S$
$(3, 2)$
$u(S, H) = \sqrt{6}$
Hotdogs (h)
Soda (s)