# Practice Exam 2 Answer Key (Problems)

*Ryan Safner*

12. You have a firm with the following production function:

$$q = kl$$

a. **In the short run, the firm has 5 units of capital. Write out the short-run production function.**

---

Plug $k = 5$ into the production function:

$$q = 5l$$

---

b. **Write down the total product, marginal product, and average product of labor for 0, 1, 2, 3, 4, and 5 workers.**

---

Make a table:

| $l$ | $q = 5l$ | $MP_l = q_2 - q_1$ | $AP_l = q/l$ |
|---|---|---|---|
| 0 | 0 | – | – |
| 1 | 5 | 5 | 5 |
| 2 | 10 | 5 | 5 |
| 3 | 15 | 5 | 5 |
| 4 | 20 | 5 | 5 |
| 5 | 25 | 5 | 5 |

---

c. **In the long run, the marginal products of labor and capital, respectively, are:**

$$MP_l = k$$
$$MP_k = l$$

**Suppose your firm needs to produce 1,000 units of output, the price of labor is \$20, and the price of capital is \$50. Find the optimal (i.e. cost-minimizing) combination of labor and capital that produces 1,000 units.**

---

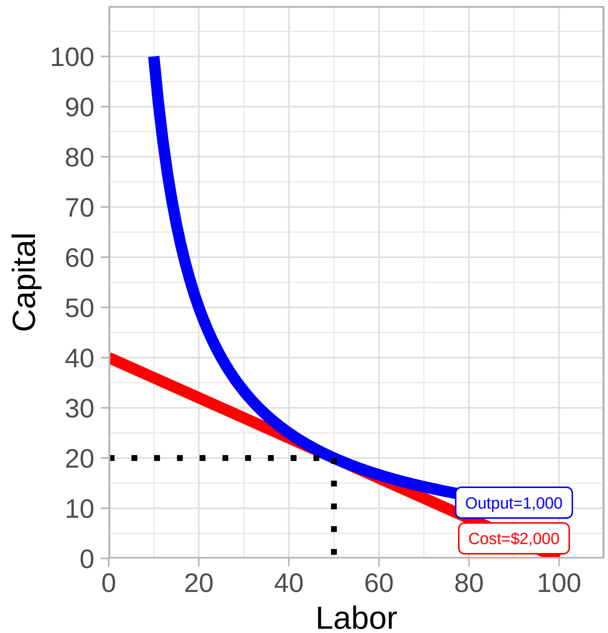
First, recognize that at the optimum, the slope of the isoquant curve (left) is equal to the slope of the isocost line:

$$\begin{aligned}\frac{MP_l}{MP_k} &= \frac{w}{r}\\ \frac{k}{l} &= \frac{10}{40}\\ \frac{k}{l} &= 0.4\\ k &= 0.4l\end{aligned}$$

Now we take this and plug it into the production function, which has to produce 1,000 units.

$$\begin{aligned} q &= kl \\ 1000 &= (0.4l)l \\ 1000 &= 0.4l^2 \\ 2500 &= l^2 \\ 50 &= l \end{aligned}$$

If $l^* = 50$, then $k^* = 0.4(50) = 20$. Thus, the cost-minimizing combination of labor and capital that produces 1,000 units of output is 50 units of labor and 20 units of capital.




d. **What is the total cost of producing with the optimal combination?**

---

$$\begin{aligned} wl + rk &= C \\ 20(50) + 50(20) &= C \\ 1000 + 1000 &= C \\ 2000 &= C \end{aligned}$$

The total cost of using 20 workers and 50 units of capital at current input prices is \$2,000.

---

e. **Graph the isoquant, isocost line, and optimum on the graph above.**

---

It will help to find the isocost line equation to graph (by solving for k):

$$\begin{aligned} wl + rk &= C \\ 20l + 50k &= 2000 \\ 50k &= 2000 - 20l \\ k &= 40 - 0.4l \end{aligned}$$

Alternatively, just calculate and graph the endpoints:

- If the firm only used capital, for \$2,000 it could use $\frac{2000}{50} = 40$
- If the firm only used labor, for \$2,000 it could use $\frac{2000}{20} = 100$

---

f. **In the long run, does this production function exhibit constant, increasing, or decreasing returns to scale?**

---

Suppose for example, we have 1 L and 1 K:

$$q = (1)(1) = 1$$

Now we double inputs to 2 L and 2 K:

$$q = (2)(2) = 4$$

Output has quadrupled, from 1 to 4 units, when we have doubled inputs from 1 to 2 K & L, so we have increasing returns to scale.

Even simpler, since this is a Cobb-Douglas function, adding the exponents

$$q = l^1 k^1$$

leads to 1+1=2, which is greater than 1, so it is again, increasing returns to scale.

---

13. Suppose you are a restaurant operating in the very competitive D.C. brunch market. You have a cost structure as follows, where $q$ is hundreds of meals served per day.

$$\begin{aligned} C(q) &= 2q^2 + 4q + 18 \\ MC(q) &= 4q + 4 \end{aligned}$$

a. Write the equations for (i) fixed costs, (ii) variable costs, (iii) average fixed costs, (iv), average variable costs, and (v) average (total) costs.

---

Fixed costs are where costs don't change with output, so any term(s) where there is no variable $q$ in them: 18 We could also see that if $q = 0$:

$$\begin{aligned} C(q) &= 2q^2 + 4q + 18 \\ C(0) &= 2(0)^2 + 4(0) + 18 \\ C(0) &= 18 \end{aligned}$$

$f = 18$

Variable costs change with output, so any term(s) with a variable $q$ in them:

$$VC(q) = 2q^2 + 4q$$

Average fixed costs:

$$\begin{aligned} AFC(q) &= \frac{FC}{q} \\ &= \frac{18}{q} \end{aligned}$$

Average variable costs:

$$\begin{aligned} AVC(q) &= \frac{VC(q)}{q} \\ &= \frac{2q^2 + 4q}{q} \\ &= 2q + 4 \end{aligned}$$

Average (total) costs:

$$\begin{aligned} AC(q) &= \frac{C(q)}{q} \\ &= \frac{2q^2 + 4q + 18}{q} \\ &= 2q + 4 + \frac{18}{q} \end{aligned}$$

Alternatively, we know that:

$$
\begin{aligned}
AC(q) &= AVC(q) + AFC(q) \\
&= (2q+4) + (\frac{18}{q}) \\
&= 2q + 4 + \frac{18}{q}
\end{aligned}
$$

---

b. **The market price is currently \$12. Calculate the profit-maximizing quantity of output.**

---

In a perfectly competitive market, optimal quantity is where $p = MR(q) = MC(q)$.

$$
\begin{aligned}
p &= MC(q) \\
12 &= 4q + 4 \\
8 &= 4q \\
2 &= q^*
\end{aligned}
$$

---

c. **At the profit-maximizing quantity, calculate the average cost.**

---

We know the average cost function from (a), just plug in the quantity we just found, 2.

$$
\begin{aligned}
AC(q) &= 2q + 4 + \frac{18}{q} \\
AC(2) &= 2(2) + 4 + \frac{18}{(2)} \\
AC(2) &= 4 + 4 + 9 \\
AC(2) &= 17
\end{aligned}
$$

---

d. **At the profit-maximizing price and quantity, calculate the total profit. Should this firm stay or exit the market in the long run?**

---

$$
\begin{aligned}
\pi &= (p - AC(q))q \\
\pi &= (12 - 17)2 \\
\pi &= (-5)2 \\
\pi &= -10
\end{aligned}
$$

Since the firm is earning losses, it will want to *exit* in the *long run*.

---

e. **Should this firm produce or shut down in the short run?**

---

Find the shut-down price, the minimum of AVC, where $AVC = MC$

$$
\begin{aligned}
AVC(q) &= MC(q) \\
2q + 4 &= 4q + 4 \\
q &= 0
\end{aligned}
$$

Plug in 0 into either $MC(q)$ or $AVC(q)$ to get a price of \$4. This is the shut down price. Since the current market price (12) is higher than the shut down price, the firm should *not* shut down in the short run, even though it is earning losses!

---

f. **What price would the firm need to charge in order to break even?**

---

Find the break-even price, the minimum of AC, where $AC = MC$

$$\begin{aligned} AC(q) &= MC(q) \\ 2q + 4 + \frac{18}{q} &= 4q + 4 \\ 2q + \frac{18}{q} &= 4q \\ \frac{18}{q} &= 2q \\ 18 &= 2q^2 \\ 9 &= q^2 \\ 3 &= q \end{aligned}$$

This is the quantity where average cost is minimized. Find what this cost is by plugging in 3 into either $MC$ or $AC$:

$$\begin{aligned} MC(q) &= 4q + 4 \\ MC(3) &= 4(3) + 4 \\ MC(3) &= 16 \end{aligned}$$

At a price of \$16, the firm breaks even ($\pi = 0$).

---

g. **In the long run, what must the equilibrium market price be for this industry, and why?**

---

For a competitive market to be in long run equilibrium, all firms must earn an economic profit of \$0. The price must be equal to average cost (the break even price) for this to happen, which again is at \$16.

Visualize this problem below: